Замена переменных Приняв $u$ и $v$ за новые независимые переменные, преобразовать следующие уравнения

1. $\dfrac{\partial^2 z}{\partial x^2} - \dfrac{\partial^2 z}{\partial y^2} = 0$ , $u = x - y, v = x + y$

2. $\dfrac{\partial^2 z}{\partial x^2} + y\dfrac{\partial^2 z}{\partial y^2} + \dfrac{1}{2}\dfrac{\partial z}{\partial x} = 0 (y > 0), u = x, v = 2\sqrt{y}$

3. $(1 + x^2)\dfrac{\partial^2 z}{\partial x^2} + (1 + y^2)\dfrac{\partial^2 z}{\partial y^2} + x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 0, u = \ln(x + \sqrt{1 + x^2})$, $v = \ln(y + \sqrt{1 + y^2})$

4. $2\dfrac{\partial^2 z}{\partial x^2} - 2\dfrac{\partial^2 z}{\partial x \partial y} + 5\dfrac{\partial z^2}{\partial y^2} + \dfrac{\partial z}{\partial x} = 0, u = \dfrac{1}{3}(x - y), v = \dfrac{1}{3}(2x + y)$

5. $\dfrac{\partial^2 z}{\partial x^2} + 2\dfrac{\partial^2 z}{\partial x \partial y} - 3\dfrac{\partial z^2}{\partial y^2} + 2\dfrac{\partial z}{\partial x} + 6\dfrac{\partial z}{\partial y} = 0, u = x + y, v = 3x - y$

6. $\dfrac{\partial^2 z}{\partial x^2} + 4\dfrac{\partial^2 z}{\partial x \partial y} + 5\dfrac{\partial z^2}{\partial y^2} + \dfrac{\partial z}{\partial x} + 2\dfrac{\partial z}{\partial y} = 0, u = 2x - y, v = x$

7. $\dfrac{\partial^2 z}{\partial x^2} - 4\dfrac{\partial^2 z}{\partial y^2} = 0, y = \dfrac{u + v}{2}, x = \dfrac{u - v}{4}$

8. $x^2\dfrac{\partial^2 z}{\partial x^2} - y^2\dfrac{\partial^2 z}{\partial y^2} - 2y\dfrac{\partial z}{\partial y}, u = xy, v = \dfrac{y}{x}$

9. $x^2\dfrac{\partial^2 z}{\partial x^2} - 2x\dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial^2 z}{\partial y^2}, u = xe^y, v = y$

10. $x^2\dfrac{\partial^2 z}{\partial x^2} - 2xy\dfrac{\partial^2 z}{\partial x \partial y} - 3y^2\dfrac{\partial^2 z}{\partial y^2}, u = \dfrac{y}{x}, v = yx^3$

11. $x^2\dfrac{\partial^2 z}{\partial x^2} + 2xy\dfrac{\partial^2 z}{\partial x \partial y} + y^2\dfrac{\partial^2 z}{\partial y^2}, u = \dfrac{y}{x}, v = y$

12. $\dfrac{\partial^2 z}{\partial x^2} - 2\sin x \dfrac{\partial^2 z}{\partial x \partial y} + (2 - \cos^2 x)\dfrac{\partial^2 z}{\partial y^2} = 0, u = x, v = y - \cos x$

13. $tg^2 x \dfrac{\partial^2 z}{\partial x^2} - 2y\ tgx \dfrac{\partial^2 z}{\partial x \partial y} + y^2 \dfrac{\partial^2 z}{\partial y^2} + \ tg^3 x \dfrac{\partial z}{\partial x} = 0, u = y \sin x, v = y$

14. $x\dfrac{\partial^2 z}{\partial x^2} - 2\sqrt{xy}\dfrac{\partial^2 z}{\partial x \partial y} + y\dfrac{\partial^2 z}{\partial y^2} + \dfrac{1}{2}\dfrac{\partial z}{\partial x} = 0, u = \sqrt{y} + \sqrt{x}, v = \sqrt{x}$

15. $y^2 \dfrac{\partial^2 z}{\partial x^2} + 2y\sqrt{xy}\dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial^2 z}{\partial y^2} = 0, y = v, x = \dfrac{u + v^2}{2}$

16. $\dfrac{\partial^2 z}{\partial x^2} - y\dfrac{\partial^2 z}{\partial y^2} = 0, y > 0,\ x = \dfrac{u + v}{2},\ y = \dfrac{(v - u)^2}{16}$

17. $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} + m^2 z = 0, 2x = u^2 + v^2, y = uv$

18. $2\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial x \partial y} - \dfrac{\partial^2 z}{\partial y^2} + \dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} = 0, u = x + 2y + 2, v = x - y - 1$

19. $(1 + x^2)\dfrac{\partial^2 z}{\partial x^2} + (1 + y^2)\dfrac{\partial^2 z}{\partial y^2} + x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 0, u = \ln(x + \sqrt{1 + x^2}),$
$v = \ln(y + \sqrt{1 + y^2})$

20. $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} + m^2 z = 0, x = e^u \cos v, v = e^u \sin v$

21. $x^2 \dfrac{\partial^2 z}{\partial x^2} - y^2 \dfrac{\partial^2 z}{\partial x \partial y} = 0, u = xy, v = \dfrac{x}{y}$

22. $x^2 \dfrac{\partial^2 z}{\partial x^2} - (x^2 + y^2)\dfrac{\partial^2 z}{\partial x \partial y} + y^2 \dfrac{\partial^2 z}{\partial y^2} = 0, u = x + y, v = \dfrac{1}{x} + \dfrac{1}{y}$

23. $x^2 \dfrac{\partial^2 z}{\partial x^2} - 2x \sin y \dfrac{\partial^2 z}{\partial x \partial y} + \sin^2 y \dfrac{\partial^2 z}{\partial y^2} = 0, u = x \operatorname{tg} \dfrac{y}{2}, v = x$

24. $\dfrac{\partial^2 z}{\partial x \partial y} = \left(1 + \dfrac{\partial z}{\partial y}\right)^3 = 0, u = x, v = y + z$

25. $x\dfrac{\partial^2 z}{\partial x^2} - y\dfrac{\partial^2 z}{\partial y^2} = 0, (x > 0, y > 0), x = (u + v)^2, y = (u - v)^2$